

Two-loop Radiative Neutrino Mechanism in an $SU(3)_L \times U(1)_N$ Gauge Model *

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By using the $L_e - L_\mu - L_\tau$ symmetry, we construct an $SU(3)_L \times U(1)_N$ gauge model that provides two-loop radiative neutrino masses as well as one-loop radiative neutrino masses. The generic smallness of two-loop neutrino masses leading to Δm_\odot^2 compared with one-loop neutrino masses leading to Δm_{atm}^2 successfully explains $\Delta m_{atm}^2 \gg \Delta m_\odot^2$ by invoking the $L_e - L_\mu - L_\tau$ breaking. The Higgs scalar (h^+) that initiates radiative mechanisms is unified into a Higgs triplet together with the standard Higgs scalar (ϕ^+, ϕ^0) to form (ϕ^+, ϕ^0, h^+) , which calls for three families of lepton triplets: $(\nu_L^i, \ell_L^i, \omega_L^i)$ ($i = 1, 2, 3$), where ω^i denote heavy neutral leptons. The two-loop radiative mechanism is found possible by introducing a singly charged scalar, which couples to $\ell_R^i \omega_R^j$ ($i, j = 2, 3$).

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I. INTRODUCTION

Atmospheric neutrino oscillations have been experimentally confirmed by the SuperKamiokande collaboration [1,2] and solar neutrinos have also been considered to be oscillating [3]. These oscillation phenomena indicate new phenomenology due to massive neutrinos [4] that requires new interactions beyond the conventional interactions in the standard model. Neutrinos can acquire masses through either seesaw mechanism [5] or radiative mechanism [6–8]. Among others, the radiative mechanism of the Zee type [6] has recently received much attention [9]. The resulting structure of neutrino mass matrix on the basis of three flavors is characterized by three off-diagonal elements. It is, then, found that in order to account for atmospheric neutrino oscillations by the radiative mechanism, neutrinos will exhibit bimaximal mixing [10,11]. This feature can be rephrased that there is a conservation of $L_e - L_\mu - L_\tau$ [12], where L_e (L_μ or L_τ) stands for the electron (muon or tau) number.

The radiative mechanism of the Zee type utilizes

1. a lepton-number-violating charged lepton (ℓ_L)-neutrino (ν_L) interaction,
2. an $SU(2)_L$ -singlet charged Higgs scalar (h^+), and
3. a second $SU(2)_L$ -doublet Higgs scalar (ϕ'),

which participate in one-loop diagrams to generate Majorana neutrino masses. The simplest extension of the standard gauge group, $SU(2)_L \times U(1)_Y$, so as to include these extra ingredients, especially for h^+ , is to employ an $SU(3)_L \times U(1)_N$ gauge group [13]. The charged Higgs scalar, h^+ , can be identified with the third member of an $SU(3)_L$ -triplet

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Higgs scalar and is unified in to (ϕ^+, ϕ^0, h^+) together with the standard Higgs doublet (ϕ^+, ϕ^0) [14]. It is also known that another advantage to use $SU(3)_L \times U(1)_N$ lies in the fact that three families of quarks and leptons are predicted if the anomaly free condition on $SU(3)_L \times U(1)_N$ and the asymptotic free condition on $SU(3)_c$ are imposed. These plausible properties have pushed us to examine how to implement one-loop radiative mechanism within the $SU(3)_L \times U(1)_N$ framework [14–17].

In this report, we further study radiative mechanism based on two-loop diagrams [18] in an $SU(3)_L \times U(1)_N$ model with (ϕ^+, ϕ^0, h^+) as a Higgs triplet [19], which in turn requires three families of heavy neutral leptons to be denoted by ω^i ($i = 1, 2, 3$) [20], which are contained in lepton triplets of $(\nu_L^i, \ell_L^i, \omega_L^i)$. It is anticipated that the interactions described by one- and two-loop diagrams can be used to generate the observed atmospheric and solar neutrino oscillations characterized by $\Delta m_{atm}^2 \gg \Delta m_\odot^2$ as have been stressed in Ref. [21]. Namely, one-loop radiative mechanism controls atmospheric neutrino oscillations while further suppressed effects from two-loop radiative mechanism provide solar neutrino oscillations. To activate two-loop radiative mechanism, an extra scalar called k^+ is introduced, which will couple to a lepton-number-violating $\ell_R\text{-}\omega_R$ [22] and $\rho'^{\dagger}\bar{\eta}^0 k^+$, where ρ'^+ and $\bar{\eta}^0$ are to be introduced in Eq.(3).

II. MODEL

The present $SU(3)_L \times U(1)_N$ gauge model is specified by the $U(1)_N$ quantum number, $N/2$, which is related to the hypercharge, Y , as $Y = -\lambda^8/\sqrt{3} + N$. The electric charge, Q_{em} , is, thus, given by $Q_{em} = (\lambda^3 + Y)/2$, where λ^a is the $SU(3)$ generator with $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ ($a, b = 1 \sim 8$). The particle content can be summarized as follows:

$$\psi_L^i = (\nu^i, \ell^i, \omega^i)_L^T : (\mathbf{3}, -\frac{1}{3}), \quad \ell_R^i : (\mathbf{1}, -1), \quad \omega_R^i : (\mathbf{1}, 0), \quad (1)$$

for leptons, where ω^i stand for three neutral heavy leptons,

$$\begin{aligned} Q_L^1 &= (u^1, d^1, u'^1)_L^T : (\mathbf{3}, \frac{1}{3}), \quad Q_L^{i=2,3} = (d^i, -u^i, d'^i)_L^T : (\mathbf{3}^*, 0), \\ u_R^{1,2,3} &: (\mathbf{1}, \frac{2}{3}), \quad d_R^{1,2,3} : (\mathbf{1}, -\frac{1}{3}), \quad u'_R : (\mathbf{1}, \frac{2}{3}), \quad d'^{2,3}_R : (\mathbf{1}, -\frac{1}{3}). \end{aligned} \quad (2)$$

for quarks. The values in the parentheses denote quantum numbers based on the $(SU(3)_L, U(1)_N)$ - symmetry. It is obvious that the pure $SU(3)_L$ anomaly vanishes since there is an equal number of triplets of quarks and leptons and antitriplets of quarks. Other anomalies are also cancelled. Our Higgs scalars are given by

1. three triplets (η, ρ, χ) that provides three members of the quark triplets as well as ordinary and heavy leptons, and one duplicate of ρ (ρ') that initiates one-loop radiative mechanism as advocated by Ref. [23],

$$\begin{aligned} \rho &= (\rho^+, \rho^0, \bar{\rho}^+)^T : (\mathbf{3}, \frac{2}{3}), \quad \rho' = (\rho'^+, \rho'^0, \bar{\rho}'^+)^T : (\mathbf{3}, \frac{2}{3}), \\ \eta &= (\eta^0, \eta^-, \bar{\eta}^0)^T : (\mathbf{3}, -\frac{1}{3}), \quad \chi = (\bar{\chi}^0, \chi^-, \chi^0)^T : (\mathbf{3}, -\frac{1}{3}), \end{aligned} \quad (3)$$

from which quarks and leptons acquire masses through their vacuum expectation values (VEV's):

$$\langle 0|\eta|0\rangle = (v_\eta, 0, 0)^T, \quad \langle 0|\rho|0\rangle = (0, v_\rho, 0)^T, \quad \langle 0|\chi|0\rangle = (0, 0, v_\chi)^T, \quad (4)$$

where the orthogonal choice of these VEV's and the absence of $\langle 0|\rho'|0\rangle$ will be guaranteed by appropriate Higgs interactions to be introduced;

2. one singly charged scalar that initiates two-loop radiative mechanism.

$$k^+ : (\mathbf{1}, 1), \quad (5)$$

which will couple to $\ell_R^i \omega_R^j$ ($i, j = 2, 3$).

The model becomes acceptable for the present discussions if it satisfies the following requirements that

- the Higgs scalar, ρ , acquire a VEV and be responsible for creating mass terms for quarks and leptons,

- the Higgs scalar, ρ' , acquire no VEV and be responsible for lepton-number-violating interactions,
- flavor-changing interactions due to Yukawa interactions well be suppressed.

Since lepton-number-violating interactions can be described by $\epsilon^{\alpha\beta\gamma}\psi_{L\alpha}^i\psi_{L\beta}^j\rho_\gamma$, which will generate the ν_L^i - ω_L^j mixing owing to $\langle 0|\rho_2|0\rangle \neq 0$, leading to a tree-level neutrino mass matrix [23]. Therefore, this coupling of ρ should be avoided in the radiative mechanism. The model contains quarks with the same charge, whose mass terms can be generated by η and χ between Q_L^1 and up-type quarks and by η^\dagger and χ^\dagger between $Q_L^{2,3}$ and down-type quarks. To achieve flavor-changing-neutral-currents (FCNC) suppression, Yukawa interactions must be constrained such that a quark flavor gains a mass from only one Higgs field [24,25].

All these restrictions can be realized by requiring interactions be invariant under a discrete transformation based on Z_4 , which is given by

$$\psi_L^{1,2,3} \rightarrow i\psi_L^{1,2,3}, \quad \ell_R^{1,2,3} \rightarrow \ell_R^{1,2,3}, \quad \omega_R^{1,2,3} \rightarrow -i\omega_R^{1,2,3}, \quad (6)$$

for leptons,

$$\begin{aligned} Q_L^1 &\rightarrow iQ_L^1, & Q_L^{2,3} &\rightarrow -iQ_L^{2,3}, & u_R^{1,2,3} &\rightarrow u_R^{1,2,3}, & d_R^{1,2,3} &\rightarrow d_R^{1,2,3}, \\ u_R^1 &\rightarrow -iu_R^1, & d_R'^{2,3} &\rightarrow id_R'^{2,3}, \end{aligned} \quad (7)$$

for quarks, and

$$\eta \rightarrow i\eta, \quad \rho \rightarrow i\rho, \quad \rho' \rightarrow -\rho', \quad \chi \rightarrow -\chi, \quad k^+ \rightarrow ik^+, \quad (8)$$

for Higgs scalars. In addition to the discrete symmetry, we also impose the $L_e - L_\mu - L_\tau \equiv L'$ conservation on our interactions to reproduce the observed atmospheric neutrino oscillations. The quantum number, L' , is assigned to be 0 for $(\eta, \rho, \rho', \chi)$, 1 for $(\psi_L^1, \ell_R^1, \omega_R^1)$, -1 for $(\psi_L^{2,3}, \ell_R^{2,3}, \omega_R^{2,3})$ and 2 for k^+ and similarly for quarks. The non-vanishing lepton number is carried by k^+ with $L=-2$ and as well as by leptons with $L=1$.

Yukawa interactions are controlled by the following lagrangian:

$$\begin{aligned} -\mathcal{L}_Y &= \frac{1}{2}\epsilon^{\alpha\beta\gamma} \sum_{i=2,3} f_{[1i]} \overline{(\psi_{\alpha L}^1)^c} \psi_{\beta L}^i \rho'_\gamma + \sum_{i=1,2,3} \overline{\psi_L^i} (f_\ell^i \rho \ell_R^i + f_\omega^i \chi \omega_R^i) \\ &+ \sum_{i,j=2,3} f_h^{ij} \overline{(\ell_R^i)^c} \omega_R^j k^+ + \overline{Q_L^1} (\eta U_R^1 + \rho D_R^1 + \chi U_R^1) \\ &+ \sum_{i=2,3} \overline{Q_L^i} (\rho^* U_R^i + \eta^* D_R^i + \chi^* D_R^i) + (\text{h.c.}), \end{aligned} \quad (9)$$

where f 's denote the Yukawa couplings and $f_{[ij]} = -f_{[ji]}$. Right-handed quarks are specified by $U_R^i = \sum_{j=1}^3 f_{uj}^i u_R^j$, $D_R^i = \sum_{j=1}^3 f_{dj}^i d_R^j$, $U_R^i = f_{u'1}^i u_R^1$ and $D_R^i = \sum_{j=2}^3 f_{d'j}^i d_R^j$. The charged lepton and heavy lepton mass matrices are taken to be diagonal. We here note that

1. the L -breaking is supplied by $\overline{(\psi_L^1)^c} \psi_L^i \rho'$ but all other interactions conserve both L and L' ,
2. the coupling of k^+ to the first family is forbidden by the L' symmetry,
3. the absence of $\chi^* D_R^i$ and $\eta^* D_R^i$ for $Q_L^{2,3}$ and of χU_R^1 and ηU_R^1 for Q_L^1 ensures the suppression of FCNC.

The Higgs interactions are described by self-Hermitian terms composed of $\phi_\alpha \phi_\beta^\dagger$ ($\phi = \eta, \rho, \chi, k^+$) and by the non-self-Hermitian terms in

$$V_0 = \lambda_0 \epsilon^{\alpha\beta\gamma} \eta_\alpha \rho_\beta \chi_\gamma + \lambda_1 (\rho^\dagger \eta) (\chi^\dagger \rho') + \lambda_2 (\rho^\dagger \rho') (\chi^\dagger \eta) + (\text{h.c.}), \quad (10)$$

which conserves both L and L' , where $\lambda_{0,1,2}$ represent coupling constants. To account for solar neutrino oscillations, the breaking of the L' -conservation should be included and is assumed to be furnished by

$$V_b = \mu_b \rho'^\dagger \eta k^+ + (\text{h.c.}), \quad (11)$$

which also breaks the L -conservation, where μ_b represents a breaking scale of the L' -conservation. The orthogonal choice of VEV's of η, ρ and χ as in Eq.(4) is supported by V_0 if $\lambda_0 < 0$. It is because V_0 gets lowered if η, ρ and χ develop VEV's. So, one can choose VEV's such that $\langle 0|\eta_1|0\rangle \neq 0$, $\langle 0|\rho_2|0\rangle \neq 0$ and $\langle 0|\chi_3|0\rangle \neq 0$. However, the similar coupling of $\eta \rho' \chi$ is forbidden by the Z_4 symmetry; therefore, the requirement of $\langle 0|\rho'|0\rangle = 0$ is not disturbed. Also forbidden is the dangerous term of $\rho^\dagger \rho'$ that would induce a non-vanishing VEV of ρ' . Finally, it should be noted that the L - and/or L' -breakings are supplied by the $\psi \psi \rho'$ - and $\rho' \eta k^+$ -terms but all others conserve both L and L' .

III. RADIATIVE NEUTRINO MASSES

The explicit form of Yukawa interactions relevant for the radiative neutrino mechanism is given by

$$\begin{aligned} \sum_{i=2,3} \left\{ f_{[1i]} \left[\left(\overline{\ell_R^{c1}} \nu_L^i - \overline{\nu_R^{1c}} \ell_L^i \right) \bar{\rho}'^+ + \left(\overline{\omega_R^{c1}} \ell_L^i - \overline{\ell_R^{c1}} \omega_L^i \right) \rho'^+ \right. \right. \\ \left. \left. + \left(\overline{\nu_R^{1c}} \omega_L^i - \overline{\omega_R^{1c}} \nu_L^i \right) \rho^0 \right] - f_h^{ij} \overline{\ell_L^{ci}} k^+ \omega_R^j \right\} - f_{\ell i}^i \left(\overline{\nu_L^i} \ell_R^i \rho^+ + \overline{\ell_L^i} \ell_R^i \rho^0 \right) \\ - f_{\omega i}^i \left(\overline{\nu_L^i} \omega_R^i \chi^- + \overline{\omega_L^i} \omega_R^i \chi^0 \right) + (\text{h.c.}) \end{aligned} \quad (12)$$

The combined use of these interactions with V_0 yields one-loop diagrams for L' -conserving Majorana neutrino mass terms as shown in Fig.1 (a) and (b), which correspond to

$$\left(\eta^\dagger \psi_L^1 \right) \epsilon^{\alpha\beta\gamma} \rho_\alpha \chi_\beta \psi_{\gamma L}^{2,3} + \left(\eta^\dagger \psi_L^{2,3} \right) \epsilon^{\alpha\beta\gamma} \rho_\alpha \chi_\beta \psi_{\gamma L}^1, \quad (13)$$

leading to one-loop radiative masses to be denoted by $m_{12,13}^{(1)}$. On the other hand, the L' -violating V_b provides two-loop diagrams as depicted in Fig.2, which correspond to an effective coupling

$$\epsilon^{\alpha\beta\gamma} \epsilon^{\alpha'\beta'\gamma'} \psi_{\alpha L}^1 \rho_\beta \chi_\gamma \psi_{\alpha' L}^1 \rho_{\beta'} \chi_{\gamma'}, \quad (14)$$

leading to two-loop radiative masses to be denoted by $m_{11}^{(2)}$.

The resulting neutrino mass matrix is found to be

$$M_\nu = \begin{pmatrix} m_{11}^{(2)} & m_{12}^{(1)} & m_{13}^{(1)} \\ m_{12}^{(1)} & 0 & 0 \\ m_{13}^{(1)} & 0 & 0 \end{pmatrix}, \quad (15)$$

where $m_{ij}^{(1,2)}$ are calculated to be

$$\begin{aligned} m_{1i}^{(1)} = f_{[1i]} v_\eta v_\rho v_\chi \left[\lambda_1 \frac{m_{\ell_i}^2 F(m_{\ell_i}^2, m_{\bar{\rho}'0}^2, m_{\rho^+}^2) - m_{\ell_1}^2 F(m_{\ell_1}^2, m_{\bar{\rho}'0}^2, m_{\rho^+}^2)}{v_\rho^2} \right. \\ \left. + \lambda_2 \frac{m_{\omega_i}^2 F(m_{\omega_i}^2, m_{\rho'0}^2, m_{\chi^0}^2) - m_{\omega_1}^2 F(m_{\omega_1}^2, m_{\rho'0}^2, m_{\chi^0}^2)}{v_\chi^2} \right], \end{aligned} \quad (16)$$

$$m_{11}^{(2)} = -2 \sum_{ij=2,3} \lambda_2 f_{[1i]} f_{[1j]} f_h^{ij} m_{\ell_i} m_{\omega_j} \mu_b v_\rho v_\chi I_{two-loop}. \quad (17)$$

The mass parameters of m_{ℓ_i} ($\equiv f_{\ell_i}^i v_\rho$) and m_{ω_i} ($\equiv f_{\omega_i}^i v_\chi$) are, respectively, the mass of the i -th charged lepton and the mass of the i -th heavy neutral lepton and masses of Higgs scalars are denoted by the subscripts in terms of their fields. The function of F and the two-loop integral of $I_{two-loop}$ are, respectively, given by

$$F(x, y, z) = \frac{1}{16\pi^2} \left[\frac{x \log x}{(x-y)(x-z)} + \frac{y \log y}{(y-x)(y-z)} + \frac{z \log z}{(z-y)(z-x)} \right], \quad (18)$$

and

$$I_{two-loop} = \frac{G(m_\ell^2, m_{\bar{\rho}'0}^2, m_k^2)}{m_k^2} \cdot \frac{G(m_\omega^2, m_{\eta^0}^2, m_k^2) - G(m_\omega^2, m_{\bar{\rho}'0}^2, m_k^2)}{m_{\eta^0}^2 - m_{\bar{\rho}'0}^2} \quad (19)$$

with

$$G(x, y, z) = \frac{1}{16\pi^2} \frac{x \ln(x/z) - y \ln(y/z)}{x-y}. \quad (20)$$

The explicit form of $I_{two-loop}$ of Eq.(19) is possible to obtain if the squared mass of k^+ , m_k^2 , is much greater than the squared masses shown inside parentheses in (19), namely, $m_k^2 \gg m_{\ell^{2,3}}^2, m_{\omega^{2,3}}^2, m_{\rho^{0,+}}^2, m_{\bar{\eta}^0}^2$. The outline of its derivation is shown in the Appendix.

The neutrino oscillations are characterized by Δm_{atm}^2 and Δm_{\odot}^2 , which are given by

$$\Delta m_{atm}^2 = m_{12}^{(1)2} + m_{13}^{(1)2} (\equiv m_\nu^2), \quad \Delta m_{\odot}^2 = 2m_\nu |m_{11}^{(2)}|, \quad (21)$$

where the anticipated relation of $|m_{11}^{(2)}| \ll |m_{1i}^{(1)}|$ for $i = 2, 3$ has been used. To enhance the bimaximal structure of M_ν in Eq.(15), we assume the equality of $m_{\omega_2} = m_{\omega_3}$ together with $f_{[12]} = f_{[13]}$ in the heavy lepton sector and $m_{\omega_1} \neq m_{\omega_{2,3}}$ to yield nonvanishing radiative corrections due to the heavy-lepton-exchanges. Tiny contributions from the ordinary lepton sector yield the deviation from the bimaximal structure.

To get order of magnitude estimates of our parameters, we first use $\Delta m_{atm}^2 \sim 3 \times 10^{-3} \text{ eV}^2$ for atmospheric neutrino oscillations [1,2,27] to mainly fix the lepton-number-violating coupling of $f_{[1i]}$ ($i=2,3$). It is then shown that solar neutrino oscillations are to be characterized by $\Delta m_{\odot}^2 \sim 2 \times 10^{-10} \text{ eV}^2$. To be more specific, we use the following parameters to compute Δm_{atm}^2 and Δm_{\odot}^2 in Eq.(21):

- $\sqrt{v_\eta^2 + v_\rho^2} = v_{weak}$, where $v_{weak} = (2\sqrt{2}G_F)^{-1/2} = 174 \text{ GeV}$, since these are the sources of weak boson masses, where $v_\rho = v_{weak}$ and $v_\eta = v_\rho/10$ are taken,
- $v_\chi = 10v_{weak}$ so that exotic particles acquire sufficiently large masses,
- $m_{\omega_{2,3}} = ev_\chi$ with $m_{\omega_{2,3}} - m_{\omega_1} = m_{\omega_{2,3}}/10$, where e stands for the electromagnetic coupling,
- $m_\eta = m_{\rho'} = v_\rho$ and $m_\chi = m_k = v_\chi$,
- the following dimensionless couplings related to L - and L' -conserving interactions are kept to be order unity: $\lambda_1 = \lambda_2 = f_h^{ij} = 1$ ($i, j = 2, 3$),
- the dimensionless couplings related to L - or L' -violating interactions are considered to be small: $f_{[1i]} \ll 1$ and $\mu_b \ll v_\chi$, where $f_{[1i]} = 10^{-7}$ (to reproduce Δm_{atm}^2) and $\mu_b = ev_\chi$ are taken.

The adopted magnitude of $f_{[1i]}$ ($=10^{-7}$) can be seen from the rough estimate of $m_{1i}^{(1)}$. By noticing that $F(m_\omega^2, m_{\rho^0}^2, m_k^2) \sim F(m_\omega^2, 0, m_k^2) = \ln(m_k^2/m_\omega^2)/(16\pi^2 m_k^2)$, we obtain $m_{1i}^{(1)} \sim 5f_{[1i]} \times 10^{-4} \text{ GeV}$ ($i=2,3$). This estimate certainly gives $f_{[1i]} \sim 10^{-7}$ in order to obtain $m_{1i}^{(1)} = \sqrt{\Delta m_{atm}^2/2} \sim 4 \times 10^{-2} \text{ eV}$ for $\Delta m_{atm}^2 \sim 3 \times 10^{-3} \text{ eV}^2$. The numerical computation by the exact formula of $I_{two-loop}$ involving Eq.(26) shown in the Appendix reproduces $\Delta m_{atm}^2 = 3 \times 10^{-3} \text{ eV}^2$ and yields $\Delta m_{\odot}^2 = 2 \times 10^{-10} \text{ eV}^2$. The approximate bimaximal mixing is characterized by $\sin 2\theta = 0.97$ reflecting the contributions from the charged lepton-exchanges to $m_{1i}^{(1)}$, where $\sin 2\theta=1$ corresponds to the bimaximal mixing. One can observe that the estimated Δm_{\odot}^2 lies in the allowed region of the observed Δm_{\odot}^2 for the vacuum oscillations [3,28]. Another case of k^+ with $L' = -2$ that couples to $e_R e_R$ yields $\Delta m_{\odot}^2 \sim 2 \times 10^{-10} (m_e/m_\tau) \text{ eV}^2$, which is inconsistent with the solar neutrino oscillation data.

IV. SUMMARY

Summarizing our discussions, we have clarified how two-loop radiative mechanism is implemented in the $SU(3)_L \times U(1)_N$ model with the lepton triplets $(\nu^i, \ell^i, \omega^i)^T$, where Zee's scalar, h^+ , that has initiated radiative mechanisms is unified into (ϕ^+, ϕ^0, h^+) together with the standard Higgs doublet, (ϕ^+, ϕ^0) . The interactions respecting the $L_e - L_\mu - L_\tau (=L')$ conservation are responsible for the one-loop radiative corrections, which generate atmospheric neutrino oscillations, while solar neutrino oscillations are induced, via two-loop radiative corrections, by the explicit L' -breaking, which is supplied by Higgs interactions involving a charged scalar (k^+) with $L' = 2$, which couples to $\tau_R^- \omega_R^{2,3}$. One-loop Majorana masses receive dominant contributions from the heavy-lepton-exchanges and the bimaximal structure is preferred by the approximate degeneracy between ω^2 - and ω^3 -masses. It is, then, shown that, for $\Delta m_{atm}^2 \sim 3 \times 10^{-3} \text{ eV}^2$, Δm_{\odot}^2 of order 10^{-10} eV^2 is obtained as the vacuum solution. The approximate bimaximal structure is characterized by $\sin 2\theta = 0.97$.

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Appendix

In this Appendix, we describe the detailed evaluation of the two-loop integral of Eq.(19). The relevant integration corresponding to Fig.2 reads

$$I_{two-loop} = \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{k^2 - m_\ell^2} \frac{1}{k^2 - m_{\rho'}^2} \frac{1}{q^2 - m_\omega^2} \cdot \frac{1}{q^2 - m_\eta^2} \frac{1}{q^2 - m_{\rho'}^2} \frac{1}{(k-q)^2 - m_k^2}. \quad (22)$$

By using that

$$\frac{1}{q^2 - m_\eta^2} \frac{1}{q^2 - m_{\rho'}^2} = \frac{1}{m_\eta^2 - m_{\rho'}^2} \left(\frac{1}{q^2 - m_\eta^2} - \frac{1}{q^2 - m_{\rho'}^2} \right), \quad (23)$$

we find that

$$I_{two-loop} = \frac{J(m_\eta^2) - J(m_{\rho'}^2)}{m_\eta^2 - m_{\rho'}^2}, \quad (24)$$

where

$$J(m^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{k^2 - m_\ell^2} \frac{1}{k^2 - m_{\rho'}^2} \frac{1}{q^2 - m_\omega^2} \frac{1}{q^2 - m^2} \frac{1}{(k-q)^2 - m_k^2}. \quad (25)$$

After the integration over k is performed, we reach

$$J(m^2) = \int_0^1 dx \int_0^1 y dy \frac{i}{16\pi^2 [y(1-y)]} I_{one-loop}(m_\omega^2, m^2, M(x, y)^2) \quad (26)$$

with

$$M(x, y)^2 = \frac{m_k^2 - (m_k^2 - m_{\rho'}^2)y - (m_{\rho'}^2 - m_\ell^2)xy}{y(1-y)}, \quad (27)$$

where $I_{one-loop}$ represents the one-loop integral expressed by F of Eq.(18):

$$I_{one-loop}(a, b, c) = \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - a} \frac{1}{q^2 - b} \frac{1}{q^2 - c} = -iF(a, b, c). \quad (28)$$

Under the approximation of $m_k^2 \gg m^2, m_{\ell, \omega, \rho'}^2$, following the estimation of $J(m^2)$ shown in the Appendix of Ref. [30], we find that

$$J(m^2) = \frac{G(m_\ell^2, m_{\rho'}^2, m_k^2) G(m_\omega^2, m^2, m_k^2)}{m_k^2}, \quad (29)$$

where

$$G(x, y, z) = \frac{1}{16\pi^2} \frac{x \ln(x/z) - y \ln(y/z)}{x - y}. \quad (30)$$

Collecting these expressions, we finally obtain Eq.(19).

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Figure Captions

FIG. 1. One loop radiative diagrams for ν^1 - ν^i ($i=2,3$) via (a) charged leptons and (b) heavy leptons.

FIG. 2. Two loop radiative diagrams for ν^1 - ν^1 .

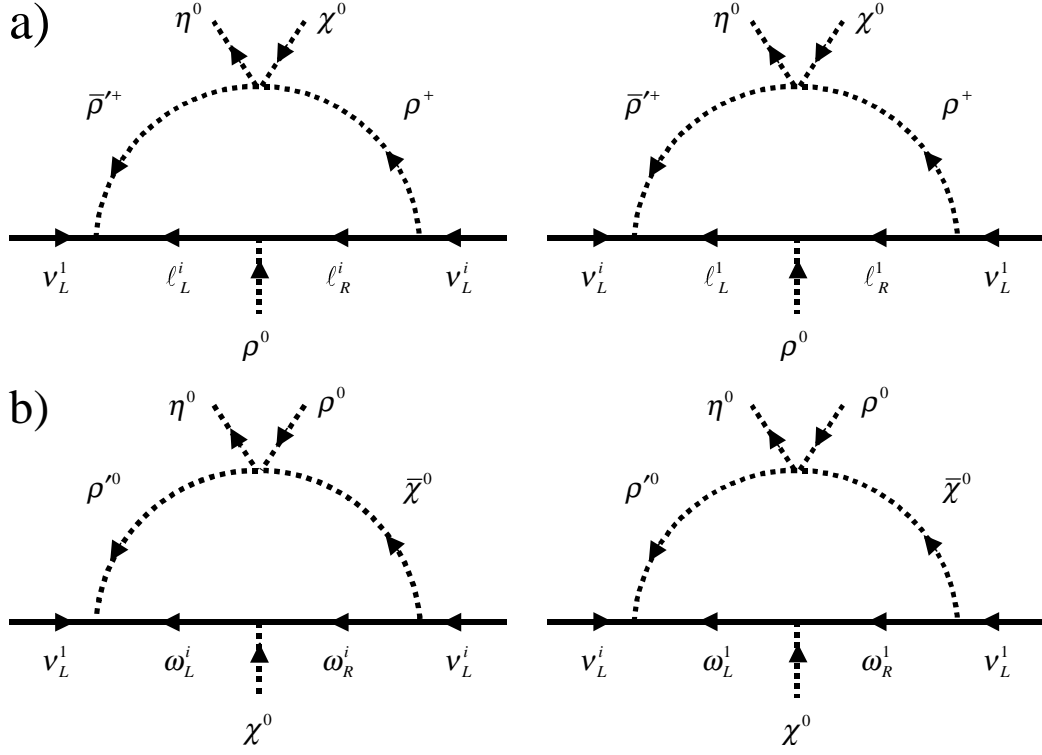


Fig. 1: One loop radiative diagrams for $\nu^1 - \nu^i$ ($i=2,3$) via (a) charged leptons and (b) heavy leptons.

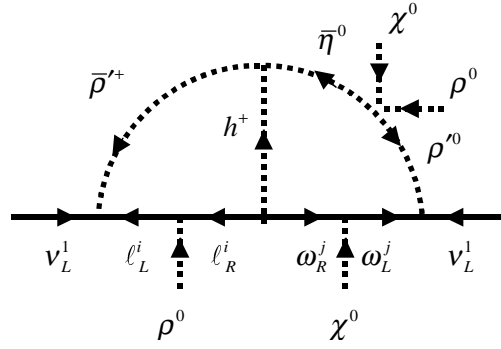


Fig. 2: Two loop radiative diagrams for $\nu^1 - \nu^1$ ($i,j=1,2$)